SOLUTION

Q.1. (A) For each of the following sub-question four alternative answers are given. Choose the correct alternative and write its alphabet:

(1) $\Delta ABC \sim \Delta PQR$ and $\angle A = 45°$, $\angle Q = 87°$, then $\angle C$ = ________ .

(a) 45° (b) 87° (c) 48° (d) 90°

(2) $\angle PQR$ is inscribed in the arc PRQ of a circle with centre 'O'. If $\angle PRQ = 75°$, then m(arc PRQ) = ________ .

(a) 75° (b) 150° (c) 285° (d) 210°

(3) A line makes an angle of 60° with the positive direction of X-axis, so the slope of a line is ________ .

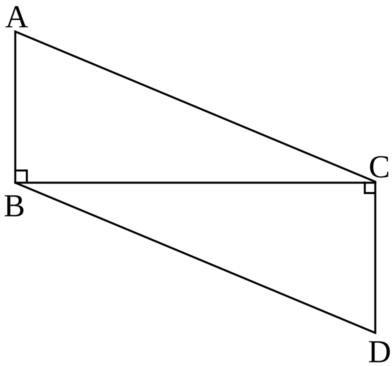
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

(4) Radius of a sector of a circle is 5 cm and length of arc is 10 cm, then the area of a sector is ________________ .

(a) $50\ cm^2$ (b) $25\ cm^2$ (c) $25\ m^2$ (d) $10\ cm^2$

Ans: (1) - (c), (2) - (d), (3) - (c), (4) - (b).

(B) Solve the following sub questions: **[4]**

(1) 

In the figure, seg AB ⊥ seg BC and seg DC ⊥ seg BC. If AB = 3 cm and CD = 4 cm, then find $\frac{A(\Delta ABC)}{A(\Delta DCB)}$.

Solution : In ΔABC and ΔDCB,

seg AB ⊥ seg BC and seg DC ⊥ seg BC ....(Given)

$\therefore$ $A(\Delta ABC) = \frac{1}{2} \times AB \times BC$ ....(1)

and $A(\Delta DCB) = \frac{1}{2} \times CD \times BC$ ....(2)

$\therefore$ $\frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{\frac{1}{2} AB \times BC}{\frac{1}{2} CD \times BC}$ From (1) and (2)

$\therefore$ $= \frac{AB}{CD}$

$\therefore$ $\boxed{\frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{3}{4}}$

(2) In cyclic □ABCD, ∠B = 75°, then find ∠D.

(3) Points A, B, C are collinear. If slope of line AB is $-\frac{1}{2}$, then find the slope of line BC.

(4) If $3\sin\theta = 4\cos\theta$, then find the value of $\tan\theta$.

Solution:

(2) In cyclic □ABCD, $\angle B = 75°$ ....(given)

$\angle B + \angle D = 180°$ ($\because$ opposite angles in a cyclic quadrilateral are supplementary)

$\therefore$ $75° + \angle D = 180°$

$\therefore$ $\angle D = 180° - 75°$

$\therefore$ $\boxed{\angle D = 105°}$

(3) Points A, B, C are collinear and the slope of line AB is $-\frac{1}{2}$. (Given)

If three points are collinear, then the slope of any two pairs of points is the same.

$\therefore$ Slope of line BC is also $\boxed{\frac{-1}{2}}$.

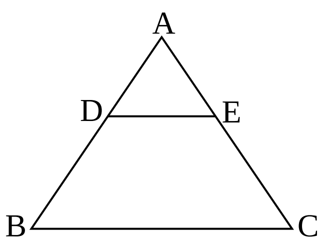
(4) $3\sin\theta = 4\cos\theta$ (Given)

$\therefore$ $\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\therefore$ $\boxed{\tan\theta = \frac{4}{3}}$

Q.2. (A) Complete the following activities and rewrite it (any *two*): [4]

(1)



In ΔABC, seg DE || side BC. If AD = 6 cm, DB = 9 cm, EC = 7.5 cm, then complete the following acitivity to find AE.

Activity: In Δ ABC, seg, DE || side BC .....(given)

$\therefore$ $\frac{AD}{DB} = \frac{AE}{EC}$ ...... $\square$

$\therefore$ $\frac{6}{9} = \frac{AE}{\square}$

$AE = \frac{6 \times 7.5}{\square}$

AE = [ ]

Solution: In ΔABC, seg, DE || side BC .....(given)

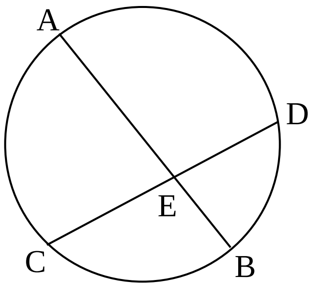
$\therefore \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$ ...... (Triangle proportionality theorem)

$\therefore \quad \dfrac{6}{9} = \dfrac{AE}{\boxed{7.5}}$

$\therefore \quad AE = \dfrac{6 \times 7.5}{\boxed{9}}$

$\therefore \quad AE = \boxed{5 \text{ cm}}$

(2)



In the above figure, chord AB and chord CD intersect each other at point E. If AE = 15, EB = 6, CE = 12, then complete the activity to find ED.

Activity:

Chord AB and chord CD intersect each other at point E .... (given)

$\therefore \quad CE \times ED = AE \times EB$ .... [ ]

$\therefore \quad \boxed{} \times ED = 15 \times 6$

$\therefore \quad ED = \dfrac{\boxed{}}{12}$

$\therefore \quad ED = \boxed{}$

Solution:

Chord AB and chord CD intersect each other at point E. (given)

$\therefore \quad CE \times ED = AE \times EB$ .... (Intersecting chords theorem)

$\therefore \quad \boxed{12} \times ED = 15 \times 6$

$\therefore \quad ED = \dfrac{\boxed{90}}{12}$

$\therefore \quad ED = \boxed{7.5}$

(3) If C(3, 5) and D(–2, –3), then complete the following acitivity to find the distance between points C and D.

Activity:

Let $C(3, 5) \equiv (x_1, y_1)$, $D(-2, -3) \equiv (x_2, y_2)$

$$\therefore \quad CD = \sqrt{(x_2 - \boxed{})^2 + (y_2 - y_1)^2} \quad \text{...... (formula)}$$

$$\therefore \quad CD = \sqrt{(-2 - \boxed{})^2 + (-3 - 5)^2}$$

$$\therefore \quad CD = \sqrt{\boxed{} + 64}$$

$$\therefore \quad CD = \sqrt{\boxed{}}$$

Solution:

Let $C(3, 5) \equiv (x_1, y_1)$, $D(-2, -3) \equiv (x_2, y_2)$

$$\therefore \quad CD = \sqrt{(x_2 - \boxed{x_1})^2 + (y_2 - y_1)^2} \quad \text{...... (formula)}$$

$$\therefore \quad CD = \sqrt{(-2 - \boxed{3})^2 + (-3 - 5)^2}$$

$$\therefore \quad CD = \sqrt{\boxed{25} + 64}$$

$$\therefore \quad CD = \sqrt{\boxed{89}}$$

(B) Solve the following sub-questions (Any *four*): [8]

(1) ΔABC ~ ΔPQR, A(ΔABC) = 81 cm², A(ΔPQR) = 121 cm². If BC = 6.3 cm, then find QR.

Solution: ΔABC ~ ΔPQR .... (given)

The ratio of the areas of two similar triangles is equal to the square of ratio of their corresponding sides.

$$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$A(\Delta ABC) = 81 \text{ cm}^2$ ....(given)

$A(\Delta PQR) = 121 \text{ cm}^2$ ....(given)

$BC = 6.3$ cm ....(given)

$$\therefore \quad \frac{81}{121} = \frac{(6.3)^2}{QR^2}$$

Taking square root on both sides,

$$\frac{9}{11} = \frac{6.3}{QR}$$

$\therefore \quad QR = \dfrac{6.3 \times 11}{9}$

$\therefore \quad \boxed{QR = 7.7 \text{ cm}}$

(2) In Δ PQR, ∠P = 60°, ∠Q = 90° and QR = $6\sqrt{3}$ cm, then find the values of PR and PQ.

Solution:

In ΔPQR, ∠P = 60°, ∠Q = 90° ....(given)

So ∠R = 30° ...(∵ sum of the three angles of the triangle is 180°)

∴ ΔPQR is 30°–60°–90° Triangle.

QR = $6\sqrt{3}$ cm ...(given)

By 30°–60°–90° triangle rule,

side opposite 30° = $\frac{1}{2}$ × hypotenuse ....(1)

side opposite 60° = $\frac{\sqrt{3}}{2}$ × hypotenuse ....(2)

Using (2), $QR = \dfrac{\sqrt{3}}{2} \times PR$

$\therefore \quad 6\sqrt{3} = \dfrac{\sqrt{3}}{2} \times PR$

$\therefore \quad PR = \dfrac{6\sqrt{3} \times 2}{\sqrt{3}}$

$\therefore \quad \boxed{PR = 12 \text{ cm}}$

Using (1), $PQ = \dfrac{1}{2} \times PR$

$\therefore \quad PQ = \dfrac{1}{2} \times 12$

$\therefore \quad \boxed{PQ = 6 \text{ cm}}$

(3) Find the slope of a line passing through the points A(2, 5) and B(4, –1).

Solution:

A (2, 5) $\equiv$ (x_1, y_1)

B (2, 5) $\equiv$ (x_2, y_2)

Slope of line AB = $\dfrac{y_2 - y_1}{x_2 - x_1}$

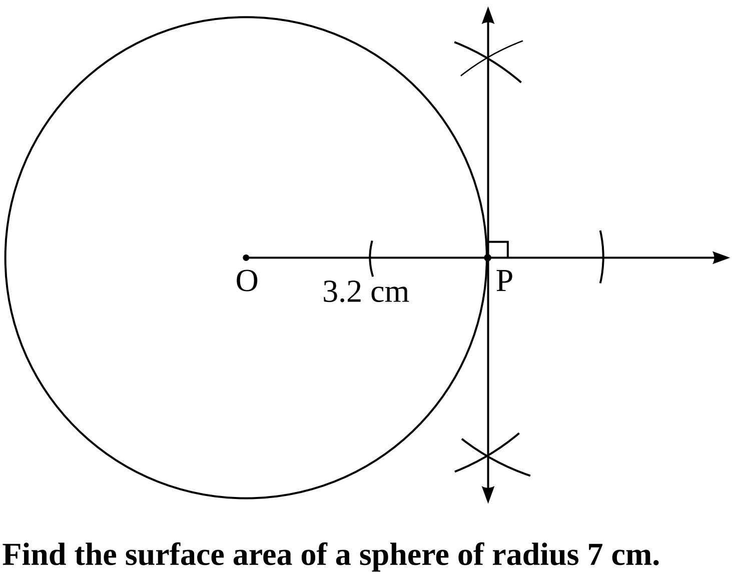
$= \dfrac{-1 - 5}{4 - 2}$

$\therefore \quad = \dfrac{-6}{2} = -3$

$\therefore$ **Slope of line AB = –3**

(4) Draw a circle with centre 'O' and radius 3.2 cm. Draw a tangent to the circle at any point P on it.

Solution:



(5) Find the surface area of a sphere of radius 7 cm.

Solution:

r = 7 cm ....(given)

Area of a sphere = ?

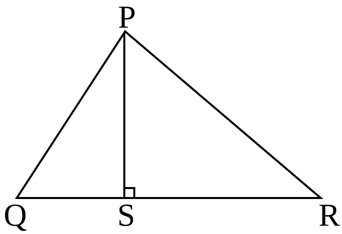
Area of sphere $= 4\pi r^2$

$= 4 \times \dfrac{22}{7} \times 7 \times 7$

$= 4 \times 22 \times 7$

$\therefore$ Area of sphere = 616 cm^2

3. (A) Complete the following activities and rewrite it. (Any one): [3]

(1) 

In ΔPQR, seg $PS \perp$ side QR, then complete the activity to prove

$PQ^2 + RS^2 = PR^2 + QS^2$

Activity:

In ΔPSQ, $\angle PSQ = 90°$

$\therefore$ $PS^2 + QS^2 = PQ^2$ ....(Pythagoras theorem)

$\therefore$ $PS^2 = PQ^2 - \square$ ....(I)

Similarly,

In ΔPSR, $\angle PSR = 90°$

$\therefore$ $PS^2 + \square = PR^2$ .... (Pythagoras theorem)

$\therefore$ $PS^2 = PR^2 - \square$ ....(II)

$\therefore$ $PQ^2 - \square = \square - RS^2$ ....from (I) and (II)

$\therefore$ $PQ^2 + \square = PR^2 + QS^2$

Solution:

In ΔPSQ, $\angle PSQ = 90°$

$\therefore$ $PS^2 + QS^2 = PQ^2$ ....(Pythagoras theorem)

$\therefore$ $PS^2 = PQ^2 - \boxed{QS^2}$ ....(I)

Similarly,

In ΔPSR, $\angle PSR = 90°$

$\therefore$ $PS^2 + \boxed{SR^2} = PR^2$ ....(Pythagoras theorem)

$\therefore$ $PS^2 = PR^2 - \boxed{SR^2}$ ....(II)

$\therefore$ $PQ^2 - \boxed{QS^2} = \boxed{PR^2} - RS^2$ ....From (I) and (II)

$\therefore$ $PQ^2 + \boxed{RS^2} = PR^2 + QS^2$

(2) Measure of arc of a circle is 36° and its length is 176 cm. Then complete the following activity to find the radius of circle.

Activity:

Here, measure of arc = $\theta = 36°$

Length of arc = $l = 176$ cm

$\therefore$ Length of arc (l) = $\frac{\theta}{360} \times \square$ ....(formula)

$\therefore$ $\square = \frac{36}{360} \times 2 \times \frac{22}{7} \times r$

$\therefore$ $176 = \frac{1}{\square} \times \frac{44}{7} \times r$

$\therefore$ $r = \frac{176 \times \square}{44}$

$\therefore$ $r = \square \times 70$

Radius of circle (r) = $\square$ cm

Solution:

Here, measure of arc = $\theta = 36°$

Length of arc = $l = 176$ cm

Length of arc (l) = $\frac{\theta}{360} \times \boxed{2\pi r}$ ....(formula)

$\therefore$ $\boxed{176} = \frac{36}{360} \times 2 \times \frac{22}{7} \times r$

$\therefore$ $176 = \frac{1}{\boxed{10}} \times \frac{44}{7} \times r$

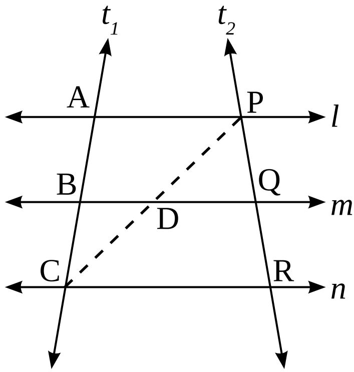
$\therefore$ $r = \frac{176 \times \boxed{70}}{44}$

$\therefore$ $r = \boxed{4} \times 70$

Radius of circle (r) = $\boxed{280}$ cm

(B) Solve the following sub-questions (Any two): [6]

(1) Prove that, "The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines."



Solution:

Given: line $l \parallel$ line m $\parallel$ line n, and t_1 and t_2 are transversals. Trasvesal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

To prove: $\frac{AB}{BC} = \frac{PQ}{QR}$

Construction: Draw seg PC, which intersects line m at point D.

Proof: In ΔACP,

$BD \parallel AP$

$\therefore \quad \frac{AB}{BC} = \frac{PD}{DC}$ ...(1) (Basic proportionality theorem)

In ΔCPR,

$DQ \parallel CR$

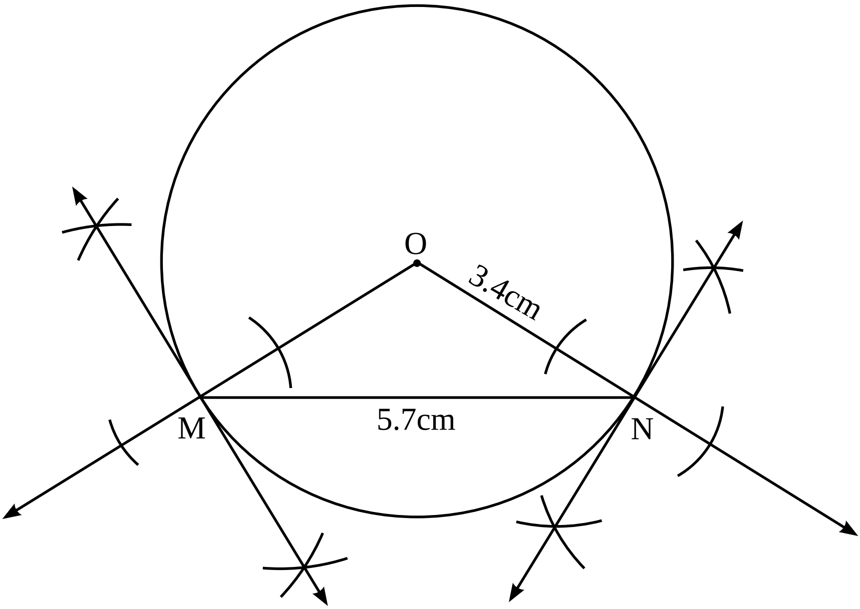
$\therefore \quad \frac{PD}{DC} = \frac{PQ}{QR}$ ...(2) (Basic proportionality theorem)

From (1) and (2), $\frac{AB}{BC} = \frac{PQ}{BC}$

Hence proved

(2) Draw a circle with centre 'O' and radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. Construct tangents at points M and N to the circle.

Solution:



(3) Prove that:

$$\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$$

Solution:

$$\text{LHS} = \frac{1}{(\sec\theta - \tan\theta)}$$

$$= \frac{(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec\theta + \tan\theta}{1}$$

$$= \sec\theta + \tan\theta$$

$\therefore$ LHS = RHS

Hence proved

(4) Radii of the top and base of frustum are 14 cm and 8 cm respectively. Its height is 8 cm. Find its curved surface area.

Solution:

$r_1 = 14$ cm

$r_2 = 8$ cm

$h = 8$ cm

slant height of a frustum (l) $= \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(14 - 8)^2 + 8^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$\therefore \quad l = 10$ cm

Curved surface area of a frustm $= \pi(r_1 + r_2)\, l$

$\therefore \quad = \frac{22}{7}(14 + 8)\, 10$

$= \frac{22 \times 22 \times 10}{7}$

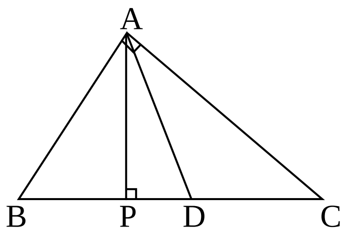
$= \frac{4840}{7}$

$= \boxed{691.42 \text{ cm}^2}$

$\therefore$ Curved surface area of the foustrum is 62.85 cm^2.

Q.4. Solve the following sub-questions (any *two*) **[8]**

(1)



In ΔABC, $\angle$BAC = 90°, seg AP $\perp$ side BC, B-P-C. Point D is the mid-point of side BC, then prove that $2AD^2 = BD^2 + CD^2$.

Solution:

Point D is the midpoint of side BC. ...(given)

$\therefore$ seg AD is the median of ΔABC.

$\therefore$ By Apollonius theorem,

$AB^2 + AC^2 = 2AD^2 + 2BD^2$ ...(1)

In ΔABC, $\angle BAC = 90°$

$\therefore$ By Pythagoras theorem,

$\therefore$ $AB^2 + AC^2 = BC^2$ ...(2)

From (1) and (2),

$2AD^2 + 2BD^2 = BC^2$

$\therefore$ $2AD^2 + 2BD^2 = (BD + CD)^2$ ...($\because$ B–D–C)

$\therefore$ $2AD^2 + 2BD^2 = BD^2 + 2BD \times BD + CD^2$

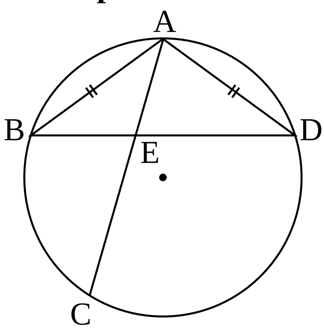
$\therefore$ $2AD^2 + 2BD^2 = BD^2 + 2BD^2 + CD^2$ ...($\because$ BD = CD)

$\therefore$ $2AD^2 = BD^2 + 2BD^2 + CD^2 - 2BD^2$

$\therefore$ $2AD^2 = BD^2 + CD^2$

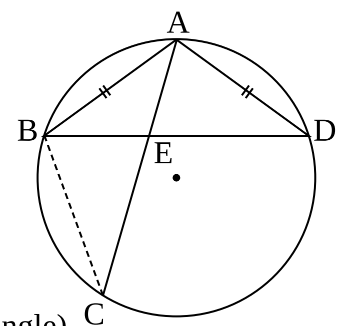
Hence proved

(2)



In the figure, chord AB ≅ chord AD. Chord AC and BD intersect each other at point E. Then prove that:

$AB^2 = AE \times AC$.



Solution:

Given: chord AB ≅ chord AD

To Prove: $AB^2 = AE \times AC$

Construction: Join points B and C.

Proof:

In ΔABC and ΔACB,

$\angle BAE = \angle BAC$ ...(Common angle)

$\because$ $\angle BEA$ and $\angle BCA$ are inscribed by same arc AB,

$\angle BEA = \angle BCA$

$\therefore$ $\Delta ABE \sim \Delta ACB$ ...(By AA test of similarity)

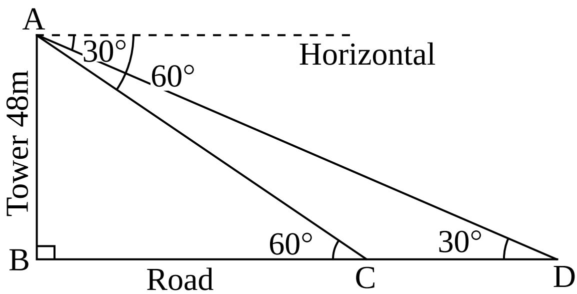
$\therefore \quad \frac{AB}{AC} = \frac{AE}{AB}$ ...(Corresponding sides of similar triangles)

$\therefore \quad \mathbf{AB^2 = AE \times AC}$

Hence proved.

(3) A straight road leads to the foot of the tower of height 48 m. From the top of the tower the angles of depression of two cars standing on the road are 30° and 60° respectively. Find the distance between the two cars.

Solution:



Let AB be the tower of height 48 m.

It is given that from the top of the tower, the angles of depression of two cars standing on the rood are 30° and 60°.

Let the first car be at point D at an angle of 30° and the second car be ar point C at an angle of 60°. (∵ Alternate angles are equal)

In ΔABC,

$\tan 60° = \frac{48}{BC}$

$\therefore \quad \sqrt{3} = \frac{48}{BC}$

$\therefore \quad BC = \frac{48}{\sqrt{3}}$ m ...(1)

In ΔABD,

$\tan 30° = \frac{AB}{BD}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{48}{BD}$

$\therefore \quad BD = 48\sqrt{3}$ ...(2)

Now, the distance between the two cars is given by BD – BC.

$$BD - BC = 48\sqrt{3} - \frac{48}{\sqrt{3}}$$

$$= \frac{48 \times 3 - 48}{\sqrt{3}}$$

$$= \frac{144 - 48}{\sqrt{3}}$$

$$= \frac{96}{\sqrt{3}}$$

$$= \frac{96}{1.73}$$

$$BD - BC = 55.49 \text{ m}$$

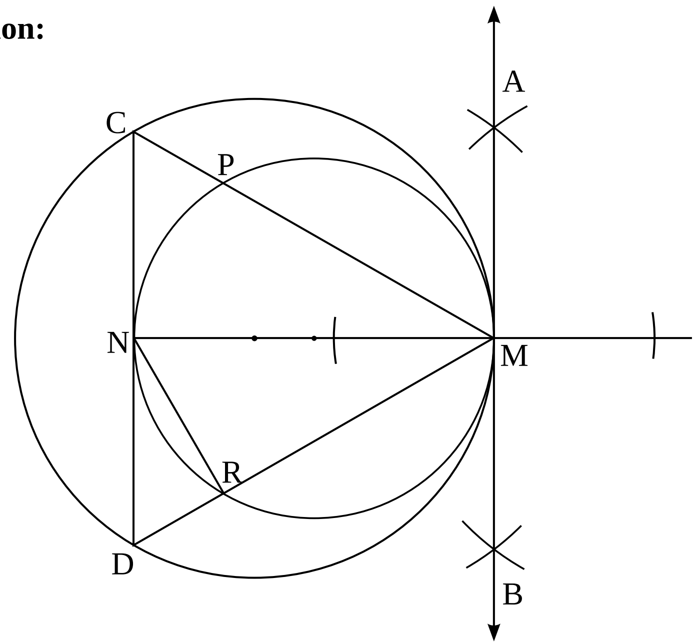
$\therefore$ **The distance between the two cars is 55.49 m.**

Q.5. Solve the following sub-questions. (Any *one*): [3]

(1) Let M be a point of contact of two internally touching circles. Let line AMB be their common tangent. The chord CD of the bigger circle touches the smaller circle at point N. The chord CM and chord DM of bigger circle intersect the smaller circle at point P and R respectively.

(a) From the above information draw the suitable figure.

Solution:



(b) Draw seg NR and seg NM and write the two pairs of congruent angles in smaller circle considering tangent and chord.

Solution:

Pairs of congruent angles in the smaller circle:

(1) $\angle CNM \cong \angle DNM$ (Each angle is of 90°.)

(2) $\angle PMN \cong \angle RMN$ ($\because$ seg MN bisects $\angle PMR$)

(c) By using the property which is used in (*b*) write the two pairs of congruent angles in the bigger circle.

Solution:

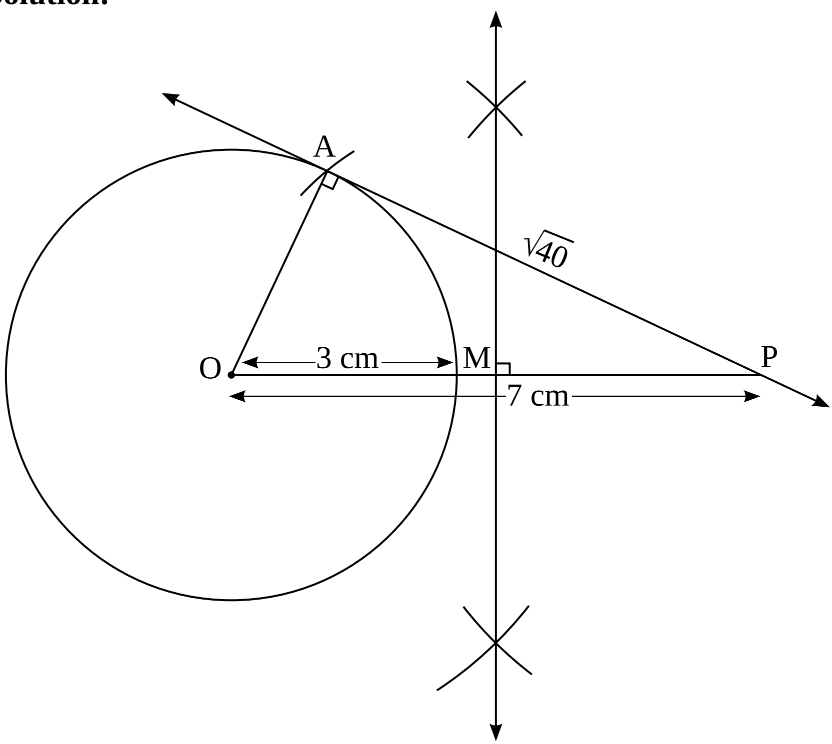
Pairs of congruent angles in the bigger circle:

(1) $\angle AMN \cong \angle BMN$ (Each angle is of 90°)

(2) $\angle CMN \cong \angle DMN$ ($\because$ seg MN bisects $\angle CMD$)

(2) Draw a circle with centre 'O' and radius 3 cm. Draw a tangent segment PA having length $\sqrt{40}$ cm from an exterior point P.

Solution:



To know the distance between centre and exterior.

In ΔOAP, $\angle OAP = 90^\circ$ (Radius-tangent point)

$\therefore$ $OP^2 = OA^2 + AP^2$ (Pythagoras theorem)

AP = tangent segment = $\sqrt{40}$ cm ....(given)

OA = radius = 3 cm ....(given)

$\therefore$ $OP^2 = 3^2 + (\sqrt{40})^2$

$= 9 + 40$

$\therefore$ $OP^2 = 49$

$\therefore$ $OP = 7$ cm

$\therefore$ The distance between the exterior point P and the centre of the circle should be 7 cm so that the length of tangent segment is $\sqrt{40}$ cm.